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Thermoelastodynamic disturbances in a half-space under the action of a buried thermal/mechanical line source

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Abstract

The transient dynamic coupled-thermoelasticity problem of a half-space under the action of a buried thermal/mechanical source is analyzed here. This situation aims primarily at modeling underground explosions and impulsively applied heat loadings near a boundary. Also, the present basic analysis may yield the necessary field quantities required to apply the Boundary Element Method in more complicated thermoelastodynamic problems involving half-plane domains. A material response for the half-space predicted by Biot's thermoelasticity theory is assumed in an effort to give a formulation of the problem as general as possible (within the confines of a linear theory). The loading consists of a concentrated thermal source and a concentrated force (mechanical source) having arbitrary direction with respect to the half-plane surface. Both thermal and mechanical line sources are situated at the same location in a fixed distance from the surface. Plane-strain conditions are assumed to prevail. Our problem can be viewed as a generalization of the classical Nakano–Lapwood–Garvin problem and its recent versions due to Payton (1968) and Tsai and Ma (1991). The initial/boundary value problem is attacked with one- and two-sided Laplace transforms to suppress, respectively, the time variable and the horizontal space variable. A 9×9 system of linear equations arises in the double transformed domain and its exact solution is obtained by employing a program of symbolic manipulations. From this solution the two-sided Laplace transform inversion is then obtained exactly through contour integration. The one-sided Laplace transform inversion for the vertical displacement at the surface is obtained here asymptotically for long times and numerically for short times.

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1. Introduction

The present work is concerned with the analysis of the problem of transient coupled thermoelastodynamic disturbances in a half-space due to the impulsive application of thermal and mechanical sources in the interior of the body. The thermal source is a buried concentrated heat flux and the mechanical source is a buried concentrated loading having different horizontal and vertical components. This situation has relevance to the case of underground nuclear explosions (Bullen and Bolt, 1987) and to the case of sudden heat loadings by impulsive electromagnetic radiation (Morland, 1968; Sve and Miklowitz, 1973; Hata, 1995).

Motions due to underground explosions or suddenly occurring earthquakes are usually modeled as elastodynamic radiation patterns due to a buried source in a half space (see e.g. Bullen and Bolt, 1987; Brady and Brown, 1990). The first theoretical investigation of this type is due to Nakano (1925) who presented a time-harmonic steady state analysis for a buried line source emitting dilatational waves in an elastic half-space. Lapwood (1949) investigated the transient version of Nakano's problem through an approximate analysis which is valid only when the depths of source and point of reception were small compared with their distance apart. The latter problem was solved in an exact manner by Garvin (1956), who employed Cagniard's (1939) technique to invert the double transforms, whereas the more general problem involving the transient action of a buried line force of arbitrary direction was considered by Payton (1968) and in full detail by Tsai and Ma (1991). The respective three-dimensional axisymmetric situation of a buried vertical point force in an elastic half-space was studied by Pekeris (1955) and Pekeris and Lifson (1957).

Most of the investigations mentioned above are considered classical work in the area of wave propagation in solids and are reviewed with much detail in the well-known treatises of Fung (1965), Eringen and Suhubi (1975), and Miklowitz (1978). Complementary work on the issue of suddenly occurring earthquakes has been presented by Knopoff and Gilbert (1960) and Burridge and Knopoff (1964) who established convenient body force equivalents for seismic dislocations. Finally, the related equilibrium (static) plane-strain problem of a line buried force in an elastic half-plane was considered by Melan (1932) and Telles and Brebbia (1981). The latter work demonstrates also that an efficient Boundary-Element formulation for half-plane problems (e.g. half-planes with cavities) can be obtained by employing the stress and displacement field due to the buried source.

In the present study, we deal with a related but more general problem than those of Nakano–Lapwood–Garvin and Payton–Tsai–Ma. The transient action of a buried line thermal and mechanical source in a half-space of a thermoelastic material is considered. As Fig. 1 depicts, a pair of vertical and horizontal forces and a concentrated heat flux act at the point ($x = 0$, $y = 0$) of the half-plane which lies at a depth H from the surface. The loadings may have an arbitrary time dependence (our analysis can deal with such cases), but here only the case of a Dirac delta variation has been worked out. Biot's (1956) coupled thermoelastodynamic theory (see also Lessen, 1956; Chadwick, 1960; Nowacki, 1971; Carlson, 1972) was employed and integral transforms were used to attack the governing equations and boundary conditions. To deal with the source terms without considering them in the pertinent field equations, we adopted a procedure introduced by Pekeris (1955) and also followed by Payton (1983) and Vardoulakis and Harnpattanapanich (1986). According to this the half-plane is separated into two regions (a half-plane region extending below the source level and an infinite-strip region extending between the source level and the surface of

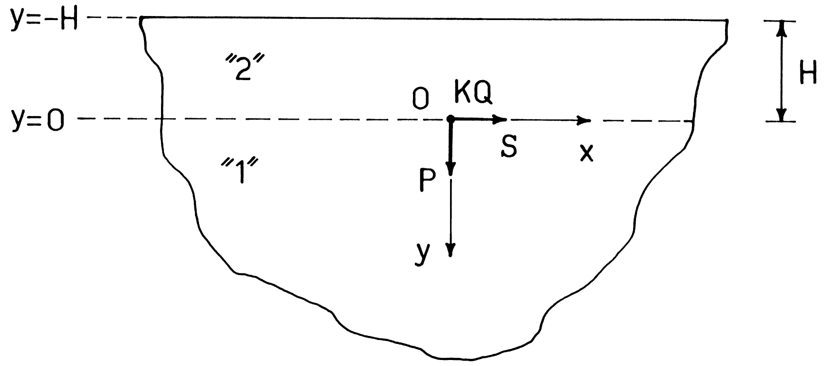

Fig. 1. Thermoelastic half-space acted upon by buried thermal and mechanical line sources at ($x = 0$, $y = 0$).

the original half-plane) with different representations of transformed displacements, stresses and temperature. Then, the solution in the double transform domain is obtained by enforcing continuity and discontinuity conditions along the source level. Here, representative numerical results were derived for the vertical displacement at the surface clearly showing the dominance of thermoelastic Rayleigh disturbances for long times. From the present basic analysis, more extensive numerical results may follow through contour integration (to invert the two-sided Laplace transform) and numerical inversion of the one-sided Laplace transform. A similar procedure was recently presented by Georgiadis et al. (1998) for a thermal fracture problem.

Finally, related work dealing with fundamental (source) solutions of dynamic coupled thermoelasticity which involves, however, infinite domains (i.e. full spaces) was carried out by Manolis and Beskos (1989) and Wang and Dhaliwal (1993). Of course, the latter problems are much simpler than the present one but these are still very useful in formulating Boundary Integral Equations, which may solve practical problems. Also, the effect of non-planarity of half-space surfaces in similar problems was studied recently by Brock et al. (1996). As regards now the mathematically similar area of poroelasticity of fluid-saturated media, work related to the present study was carried out by Vardoulakis and Harnpattanapanich (1986) and Harnpattanapanich and Vardoulakis (1987), who dealt with coupled inertialess problems of half-planes and layers under buried or surface loadings. In the latter work, the absence of inertia terms renders the problem mathematically different than the present one. We would like also to mention that in thermal-shock problems the importance of inertia (dynamic) effects was revealed in the studies of Sternberg and Chakravorty (1959a, b) and the importance of both inertia and thermal-coupling effects in the studies of Hetnarski (1961), Boley and Tolins (1962) and Francis (1972).

2. Problem statement

Consider a body occupying the half-plane ($-\infty < x < \infty, -H < y < \infty$) under plane-strain conditions. The body is of quiescent past and at uniform initial temperature (see Fig. 1). At time $t = 0$, this body is acted upon by thermal and mechanical line sources at the origin ($x = 0, y = 0$), which is situated at a depth H below the surface. The concentrated thermal loading may have an arbitrary time dependence $g_Q(t)$ and has an intensity KQ, where K is the thermal conductivity with dimensions of (power) (unit length)$^{-1}$ (°C)$^{-1}$, °C means degrees of temperature and Q is a multiplier expressed in (°C) (dimensions of $g_Q(t)$)$^{-1}$. The concentrated mechanical loading has a horizontal component $S \cdot g_S(t)$ and a vertical component $P \cdot g_P(t)$, where $g_S(t)$ and $g_P(t)$ may be arbitrary functions of time and the intensities S and P are expressed in, respectively, (force) (unit length)$^{-1}$ (dimensions of $g_S(t)$)$^{-1}$ and (force) (unit length)$^{-1}$ (dimensions of $g_P(t)$)$^{-1}$. Then, according to the linear isotropic coupled thermoelastodynamic theory of Biot (1956) (see also e.g. Lessen, 1956; Carlson, 1972) the governing equations for the plane problem described above are as follows

$$\boldsymbol{\sigma} = \mu(\nabla\boldsymbol{u}+\boldsymbol{u}\nabla)+\lambda(\nabla\cdot\boldsymbol{u})\mathbf{1}-\kappa_0(3\lambda+2\mu)\theta\mathbf{1}, \tag{1}$$

$$\boldsymbol{q} = -K\nabla\theta, \tag{2}$$

$$\mu\nabla^2\boldsymbol{u}+(\lambda+\mu)\nabla(\nabla\cdot\boldsymbol{u})-\kappa_0(3\lambda+2\mu)\nabla\theta+\boldsymbol{f}\delta(x)\cdot\delta(y) = \rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{3}$$

$$K\nabla^2\theta-\rho c_v\frac{\partial\theta}{\partial t}-\kappa_0(3\lambda+2\mu)T_0\frac{\partial(\nabla\cdot\boldsymbol{u})}{\partial t}+KQ\cdot g_Q(t)\cdot\delta(x)\cdot\delta(y) = 0, \tag{4}$$

where (1) is the Neumann–Duhamel relation, (2) is the heat conduction equation, (3) is the displacement-temperature equation of motion, and (4) is the coupled heat equation. Also, in the above equations which hold in the (x, y)-plane, $\boldsymbol{\sigma}$ is the stress tensor with components $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$, $\boldsymbol{u}$ is the displacement vector with components (u_x, u_y), $\theta = T-T_0$ is the change in temperature, T is the current temperature, T_0 is the initial temperature, $\boldsymbol{q}$ is the heat-flux vector whose components (q_x, q_y) have dimensions of (power) (unit area)$^{-1}$, (λ, μ) are the Lamé constants, κ_0 is the coefficient of linear expansion expressed in (°C)$^{-1}$, ρ is the mass density, c_v is the specific heat at constant deformation expressed in (energy) (unit mass)$^{-1}$ (°C)$^{-1}$, $\boldsymbol{f}$ is a vector having $S\cdot g_S(t)$ as its x-component and $P\cdot g_P(t)$ as its y-component, $\delta()$ denotes the Dirac delta distribution with dimensions $()^{-1}$, $\mathbf{1}$ is the identity tensor, ∇ is the gradient operator, and $\nabla^2 = (\partial^2/\partial x^2)+(\partial^2/\partial y^2)$ is the Laplace operator. All field quantities above are functions of (x, y, t).

In addition, zero initial conditions are taken, i.e.

$$\boldsymbol{u} = \partial\boldsymbol{u}/\partial t = \theta = 0 \quad \text{for } t \leqslant 0 \quad \text{in } (-\infty < x < \infty, -H < y < \infty), \tag{5}$$

and we also assume that the half-plane surface $y = -H$ is traction free and insulated (i.e. no heat is conducted through the half-plane surface and air).

The problem statement is completed now by writing the pertinent finiteness conditions at remote regions (see e.g. Duff and Naylor, 1966)

$$u_x, u_y \sim \frac{U[t-(r/V)]}{[t^2-(r/V)^2]^{1/2}} \quad \text{for } r\to\infty, \tag{6a}$$

$$\theta \sim \frac{\exp(-r^2/4kt)}{t} \quad \text{for } r\to\infty, \tag{6b}$$

where $U[\,]$ is the Heaviside unit-step function, $r = (x^2+y^2)^{1/2}$, V is a velocity-like real and positive constant, and $k = K/\rho c_v$ is the diffusivity. Thus, even in the case of heat-conduction, where temperature signals travel at an infinite speed, the field at infinity remains bounded.

Then, by following the method of Pekeris (1955) explained in the Introduction, we introduce an imaginary line along $(-\infty < x < \infty, y = 0)$ separating the original half-plane into the half-plane $(-\infty < x < \infty, 0 < y < \infty)$ (Region '1' in Fig. 1) and the strip $(-\infty < x < \infty, -H < y < 0)$ (Region '2' in Fig. 1), and consider pertinent continuity and discontinuity conditions at $y = 0$ along with the (standard) boundary conditions at $y = -H$

$$u_{x1}(x,0,t) = u_{x2}(x,0,t) \quad \text{for } -\infty < x < \infty, \tag{7a}$$

$$u_{y1}(x,0,t) = u_{y2}(x,0,t) \quad \text{for } -\infty < x < \infty, \tag{7b}$$

$$\theta_1(x,0,t) = \theta_2(x,0,t) \quad \text{for } -\infty < x < \infty, \tag{7c}$$

$$\sigma_{yy1}(x,0,t) - \sigma_{yy2}(x,0,t) = P \cdot \delta(x) \cdot g_P(t) \quad \text{for } -\infty < x < \infty, \tag{7d}$$

$$\sigma_{xy1}(x,0,t) - \sigma_{xy2}(x,0,t) = S \cdot \delta(x) \cdot g_S(t) \quad \text{for } -\infty < x < \infty, \tag{7e}$$

$$\frac{\partial \theta_1(x,0,t)}{\partial y} - \frac{\partial \theta_2(x,0,t)}{\partial y} = Q \cdot \delta(x) \cdot g_Q(t) \quad \text{for } -\infty < x < \infty, \tag{7f}$$

$$\sigma_{yy}(x,-H,t) = 0 \quad \text{for } -\infty < x < \infty, \tag{8a}$$

$$\sigma_{xy}(x,-H,t) = 0 \quad \text{for } -\infty < x < \infty, \tag{8b}$$

$$\frac{\partial \theta(x,-H,t)}{\partial y} = 0 \quad \text{for } -\infty < x < \infty, \tag{8c}$$

where the subscript '1' or '2' in a field quantity means that the line $y = 0$ is approached as $y \to 0^+$ or $y \to 0^-$, respectively. Introducing eqns (7) allows indeed to formulate the initial/boundary value problem without the explicit consideration of the source terms $\boldsymbol{f}\delta(x) \cdot \delta(y)$ and $KQ \cdot g_Q(t) \cdot \delta(x) \cdot \delta(y)$ in the field eqns (3) and (4), respectively, and therefore, leads to a considerable reduction of algebraic manipulations.

In this way, the original problem (1)–(6) and (8) can alternatively be described by (1), (2), (5)–(8) along with the following field equations (the latter being with no source terms)

$$\nabla^2 u_x + \frac{\partial[(m^2-1)\Delta + \kappa\theta]}{\partial x} - \frac{1}{V_2^2}\frac{\partial^2 u_x}{\partial t^2} = 0, \tag{9a}$$

$$\nabla^2 u_y + \frac{\partial[(m^2-1)\Delta + \kappa\theta]}{\partial y} - \frac{1}{V_2^2}\frac{\partial^2 u_y}{\partial t^2} = 0, \tag{9b}$$

$$\frac{K}{\mu}\nabla^2 \theta - \frac{c_v}{V_2^2}\frac{\partial \theta}{\partial t} + \kappa T_0 \frac{\partial \Delta}{\partial t} = 0, \tag{10}$$

where (9) and (10) follow directly from (3) and (4) by introducing the shear-wave velocity $V_2 = (\mu/\rho)^{1/2}$, the dilatation $\Delta = (\partial u_x/\partial x) + (\partial u_y/\partial y)$ and the normalized coefficient of linear expansion $\kappa = -\kappa_0(3\lambda + 2\mu)/\mu = \kappa_0(4-3m^2) < 0$, with $m = V_1/V_2 > 1$ and $V_1 = [(\lambda + 2\mu)/\rho]^{1/2}$ being the longitudinal wave velocity in the non-thermal (purely elastic) theory.

Finally, for convenience in the subsequent analysis, the time is normalized by introducing the new variable $s = V_1 t$, and also the arbitrary functions of time dependence of the sources $g_Q(t)$, $g_S(t)$ and $g_P(t)$ are replaced by the Dirac $\delta(t)$ so that any response due to a general time dependence of loading to be obtained from the present solution through convolution. In addition, when the double transformed solution (corresponding to the $\delta(t)$-loading) is obtained at a later stage, we shall identify the pertinent alterations needed to provide the solution due to an arbitrary $g_j(t)$-loading (with $j = Q, S, P$).

3. Integral-transform analysis

The problem will be attacked by means of one- and two-sided Laplace transforms. The appropriate definitions are as follows

$$\Phi(x,y,p) = \int_0^{\infty} \varphi(x,y,s)\cdot \mathrm{e}^{-ps}\,\mathrm{d}s, \quad \varphi(x,y,s) = (1/2\pi i)\int_{\Gamma_1} \Phi(x,y,p)\cdot \mathrm{e}^{ps}\,\mathrm{d}p, \tag{11a,b}$$

$$\Phi^*(q,y,p) = \int_{-\infty}^{\infty} \Phi(x,y,p)\cdot \mathrm{e}^{-pqx}\,\mathrm{d}x, \quad \Phi(x,y,p) = (p/2\pi i)\int_{\Gamma_2} \Phi^*(q,y,p)\cdot \mathrm{e}^{pqx}\,\mathrm{d}q, \tag{12a,b}$$

where for the one-sided direct transform we save a capital letter and the two-sided direct transform is denoted by an asterisk. We also notice that (van der Pol and Bremmer, 1950): (1) Because of the identity theorem for analytic functions it is sufficient to view $\Phi(x,y,p)$ as a function of a real variable p over some segment of the real axis in the half-plane of analyticity. Once $\Phi(x,y,p)$ is determined as an explicit function of p in the course of solving the transformed differential equations, the definition of $\Phi(x,y,p)$ can be extended to the whole complex p-plane, except for isolated singular points, through analytic continuation. (2) The variable q is complex. (3) The integration paths Γ_1 and Γ_2 are lines parallel to the imaginary axis in the p- and q-plane, respectively, and they lie within the regions of analyticity.

Applying now (11a) and (12a) to the governing eqns (1), (9) and (10), and considering (5) and (6) yields the following general expressions for the transformed temperature change, displacements and stresses which are different in the regions '1' and '2'.

(a) Region '1' ($0 < y < \infty$):

$$\begin{bmatrix} \dfrac{\kappa}{m^2}\Theta^* \\ pU_x^* \\ pU_y^* \\ \dfrac{1}{\mu}\Sigma_{xy}^* \\ \dfrac{1}{\mu}\Sigma_{yy}^* \\ \dfrac{1}{\mu}\Sigma_{xx}^* \end{bmatrix} = \begin{bmatrix} M_+ & M_- & 0 & 0 & 0 & 0 \\ -q & -q & 1 & 0 & 0 & 0 \\ -a_+ & a_- & \dfrac{q}{\beta} & 0 & 0 & 0 \\ -2qa_+ & -2qa_- & \dfrac{T}{\beta} & 0 & 0 & 0 \\ -T & -T & -2q & 0 & 0 & 0 \\ T_+ & T_- & 2q & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1\,\mathrm{e}^{-pa_+y} \\ X_2\,\mathrm{e}^{-pa_-y} \\ X_3\,\mathrm{e}^{-p\beta y} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{13}$$

(b) Region '2' ($-H < y < 0$):

$$\begin{bmatrix} \dfrac{\kappa}{m^2}\Theta^* \\ pU_x^* \\ pU_y^* \\ \dfrac{1}{\mu}\Sigma_{xy}^* \\ \dfrac{1}{\mu}\Sigma_{yy}^* \\ \dfrac{1}{\mu}\Sigma_{xx}^* \end{bmatrix} = \begin{bmatrix} M_+ & M_+ & M_- & M_- & 0 & 0 \\ -q & -q & -q & -q & 1 & 1 \\ -a_+ & a_+ & -a_- & a_- & -\dfrac{q}{\beta} & \dfrac{q}{\beta} \\ 2qa_+ & -2qa_+ & 2qa_- & -2qa_- & -\dfrac{T}{\beta} & \dfrac{T}{\beta} \\ -T & -T & -T & -T & -2q & -2q \\ T_+ & T_+ & T_- & T_- & 2q & 2q \end{bmatrix} \begin{bmatrix} X_4\,\mathrm{e}^{pa_+y} \\ X_5\,\mathrm{e}^{-pa_+y} \\ X_6\,\mathrm{e}^{pa_-y} \\ X_7\,\mathrm{e}^{-pa_-y} \\ X_8\,\mathrm{e}^{p\beta y} \\ X_9\,\mathrm{e}^{-p\beta y} \end{bmatrix}, \tag{14}$$

where it should be noticed that the solution (13) is bounded at $y \to \infty$ coping thus with the restrictions in (6), whereas such a constraint need not be imposed on the solution (14). In the above equations, the yet unknown $X_1, X_2, \ldots, X_9$ are arbitrary functions of (q,p) which will be determined from conditions (7) and (8) in our particular problem. Also, the following definitions were employed

$$a_+ \equiv a_+(q,p) = (m_+^2 - q^2)^{1/2}, \quad a_- \equiv a_-(q,p) = (m_-^2 - q)^{1/2}, \quad \beta \equiv \beta(q) = (m^2 - q^2)^{1/2}, \tag{15a,b,c}$$

$$m_+ = \frac{1}{2}\left[\left(1 + \frac{1}{(hp)^{1/2}}\right)^2 + \frac{\varepsilon}{hp}\right]^{1/2} + \frac{1}{2}\left[\left(1 - \frac{1}{(hp)^{1/2}}\right)^2 + \frac{\varepsilon}{hp}\right]^{1/2}, \tag{16a}$$

$$m_- = \frac{1}{2}\left[\left(1 + \frac{1}{(hp)^{1/2}}\right)^2 + \frac{\varepsilon}{hp}\right]^{1/2} - \frac{1}{2}\left[\left(1 - \frac{1}{(hp)^{1/2}}\right)^2 + \frac{\varepsilon}{hp}\right]^{1/2}, \tag{16b}$$

$$M_+ = m_+^2 - 1, \quad M_- = m_-^2 - 1, \tag{17a,b}$$

$$T_+ = 2a_+^2 - m^2, \quad T_- = 2a_-^2 - m^2, \quad T = 2\beta^2 - m^2 \equiv m^2 - 2q^2, \tag{18a,b,c}$$

with $\varepsilon = (T_0/c_v)(\kappa V_2/m)^2$ being the dimensionless coupling coefficient and $h = (KV_2/\mu m c_v)$ being the thermoelastic length, both being introduced by Chadwick (1960). He also provides numerical values for ε and h corresponding to several engineering materials which generally show that $\varepsilon = O(10^{-2})$ and $h = O(10^{-10})$ m. However, it should be mentioned at this point that Chadwick (1960) has provided a different approach (than the one presented here) based on a displacement-potential formulation. In addition, pertinent branch cuts are introduced in the complex q-plane for the functions $a_+(q,p)$, $a_-(q,p)$ and $\beta(q)$ in the manner shown for instance for $\beta(q)$ in Fig. 2 (i.e. outwards with respect to the origin $q = 0$). Their choice is consistent with the equalities

$$(m_\pm^2 - q^2)^{1/2} = \frac{1}{i}(q^2 - m_\pm^2)^{1/2}, \quad (m^2 - q^2)^{1/2} = \frac{1}{i}(q^2 - m^2)^{1/2}, \tag{19a,b}$$

and their usage will become apparent in the course of inverting the two-sided Laplace transform.

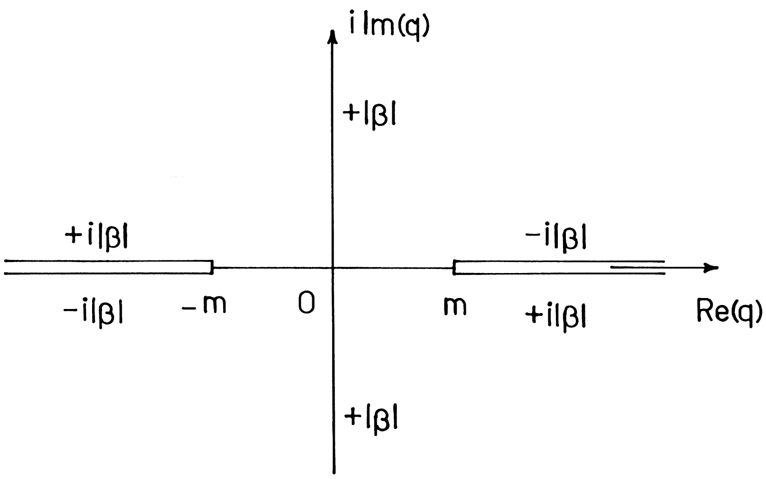

Fig. 2. The cut complex q-plane for the function $\beta(q) = (m^2 - q^2)^{1/2}$. Similar branch cuts, emanating from the points $m_+(p)$ and $m_-(p)$, are also introduced for, respectively, the functions $a_+(q,p)$ and $a_-(q,p)$.

Finally, in view of the definitions in (16), the following inequalities can be proven

$$m_- < m_+ < m \quad \text{for } hp > \frac{m^2(1+\varepsilon)-1}{m^2(m^2-1)}, \tag{20a}$$

$$m_- < m < m_+ \quad \text{for } hp < \frac{m^2(1+\varepsilon)-1}{m^2(m^2-1)}, \tag{20b}$$

whereas Brock (1995) provides the following approximate forms which considerably simplify one-sided Laplace transform inversions

$$m_+ \cong 1, \quad m_- \cong \frac{1}{(hp)^{1/2}} \quad \text{for } \frac{s}{h} \ll 1, \tag{21a}$$

$$m_+ \cong \left(\frac{1+\varepsilon}{hp}\right)^{1/2}, \quad m_- \cong \frac{1}{(1+\varepsilon)^{1/2}} \quad \text{for } \frac{s}{h} \gg 1. \tag{21b}$$

Furthermore, it turns out that (20a) and (21a) hold true only during a very small initial time-interval of the process which for most materials is $t < O(10^{-14}\text{ s})$. In the present study, however, information is needed generally for longer times so we shall focus interest only on the case (20b) and employ (21b) appropriately. Since (21b) was obtained by considering $s \to (1/p)$ and expanding in series, any case $(s/h) \geqslant 100$ leads to a reasonable approximation for m_+ and m_-.

Now, transformation according to (11a) and (12a) of the continuity/discontinuity conditions (7) [with $g_P(t) = g_S(t) = g_Q(t) = \delta(t)$] and the boundary conditions (8) along with the general transformed solutions (13) and (14) leads to a linear algebraic system of nine equations in the nine unknown $X_1, X_2, \ldots, X_9$. Obviously, this system has to be solved in an exact manner (i.e. not numerically). Solution by hand via Gauss elimination proved rather tough, whereas use of some advanced and popular symbolic-manipulations programs proved inefficient. However, the solution was provided by the more primitive symbolic-algebra program DERIVE through successive substitutions. The rather lengthy expressions for $X_1, X_2, \ldots, X_9$ along with the original system are given in Appendix A.

Having available the solution $(X_1, X_2, \ldots, X_9)$ and therefore, by (13) and (14), the general expressions for the double transformed temperature, displacement and stresses allows determining the field quantities at any point of the original space and at any time through successive inversions of the type (12b) and (11b). Notice also that if a general dependence from time of the loading functions is to be considered [i.e. arbitrary but Laplace transformable functions $g_Q(t)$, $g_P(t)$ and $g_S(t)$ instead of $\delta(t)$], then the quantities Q, P and S in eqns (A2) of Appendix A should be replaced by, respectively, $(Q/V_1)\cdot G_Q(p)$, $(P/V_1)\cdot G_P(p)$ and $(S/V_1)\cdot G_S(p)$, where $G_j(p)$ $(j = Q, P, S)$ denote the one-sided Laplace transforms of the functions $g_j(s/V_1 \equiv t)$.

In principle, the two-sided Laplace-transform inversion (12b) can be accomplished in an exact fashion by deforming the Bromwich path $(-i\infty, i\infty)$ in the q-plane and employing standard results of complex-variable theory. In this way, the one-sided Laplace transformed expressions of interest appear in the form of integrals with semi-infinite integration intervals because of the involvement of branch cuts of the type shown in Fig. 2. Then, the one-sided Laplace-transform inversion (11b) may be obtained approximately either by asymptotic considerations and use of an argument of the Cagniard–de Hoop type (Cagniard, 1939; de Hoop, 1960) or by a numerical technique. In the present study, both approaches were utilized with their choice depending upon the time interval of interest.

In what follows, we focus attention on the normal displacement at the surface $u_y(x, y = -H, t)$ and present details for the inversions.

4. Surface displacement due to a heat source

The double transformed displacement $U_y^*(q, y = -H, p)$ for $Q \neq 0$, $P \neq 0$ and $S \neq 0$ is given in Appendix B. Results, however, will be presented here for the case $Q \neq 0$, $P = S = 0$, i.e. for a buried heat source only. The respective double transformed expression is written as

$$U_y^*(q, y = -H, p) = \frac{Q\kappa V_1}{p^2}\frac{m^2 - 2q^2}{D(q,p)}\left[(m_+^2 - q^2)^{1/2}\,\mathrm{e}^{-a_- Hp} - (m_-^2 - q^2)^{1/2}\,\mathrm{e}^{-a_+ Hp}\right], \tag{22}$$

where

$$D(q,p) \equiv D = a_- M_- R_+ - a_+ M_+ R_-, \tag{23}$$

with

$$R_+(q,p) \equiv R_+ = 4q^2 a_+ \beta + T^2, \quad R_-(q,p) \equiv R_- = 4q^2 a_- \beta + T^2, \tag{24a,b}$$

being the thermoelastic counterparts of the classical (i.e. non-thermal purely elastic) transformed Rayleigh function defined (see e.g. Achenbach (1973)) as $R(q) = 4q^2(1-q^2)^{1/2}\beta + T^2$, where T is given in (18c). Moreover, the function (D/a_+) exhibits the zeroes $q = \pm q_R$ which correspond to thermoelastic Rayleigh wavefronts propagating along the traction-free surface of the half-plane (Brock, 1995). The roots of (D/a_+) depend upon p and the thermoelastic Rayleigh waves are thus dispersive (as opposed to the elastic Rayleigh waves) having a propagation velocity $V_R(t) = V_1/q_R$. The latter, however, varies in practice only slightly with time (Georgiadis et al., 1997). A closed-form expression for the root q_R is provided by Brock (1995) as

$$q_R(p) = \frac{m^2[A+(m_-/m_+)B]^{1/2}}{C[2(m^2-1)]^{1/2}}, \tag{25}$$

where

$$A = \frac{M_+}{M_+-M_-}, \quad B = -\frac{M_-}{M_+-M_-}, \tag{26a,b}$$

$$C = \exp\left[\frac{1}{\pi}\int_{m_-}^{m} \arctan\left(\frac{\alpha_-}{a_+}\left(\frac{M_-}{M_+} - a_+\frac{4\omega^2\beta}{T^2B}\right)\right)\frac{d\omega}{\omega} + \frac{1}{\pi}\int_{m}^{m_+} \arctan\left(\frac{\alpha_-}{a_+}\left(\frac{M_+}{M_-} + a_-\frac{4\omega^2 b}{T^2A}\right)^{-1}\right)\frac{d\omega}{\omega}\right], \tag{27}$$

with

$$\alpha_- = (\omega^2-m_-^2)^{1/2}, \quad b = (\omega^2-m^2)^{1/2}, \tag{28a,b}$$

and q_R being such that the inequality $m < q_R < m_+$ always holds [of course, within the time interval implied by (20b)].

Now, according to the inversion formula in (12b), eqn (22) yields

$$U_y(x, y = -H, p) = \frac{Q\kappa V_1}{2\pi ip}(I_1 - I_2), \tag{29}$$

where I_1 and I_2 are the following complex integrals

$$I_1 = \int_{-i\infty}^{i\infty} \frac{a_+ T e^{-a_- Hp}}{D} e^{pqx}\, dq, \tag{30a}$$

$$I_2 = \int_{-i\infty}^{i\infty} \frac{a_- T e^{-a_+ Hp}}{D} e^{pqx}\, dq. \tag{30b}$$

Further, the evaluation of these integrals can be effected by using contour integration, Cauchy's integral theorem and Jordan's lemma (see e.g. van der Pol and Bremmer, 1950). We first alter the $(-i\infty, i\infty)$-path, in the way shown in Fig. 3, so as to include two large quarter-circular paths at infinity in the left half-plane $Re(q) < 0$, for the case $x \geqslant 0$ [the other possibility $x \leqslant 0$ is treated by considering similar paths in the right half-plane $Re(q) > 0$], and also branch-line paths along $(-\infty, -m_-)$. Then we proceed to the following considerations: (1) we take into account the behavior of the integrands along the branch cuts, (2) we employ Jordan's lemma for the quarter-circular paths at infinity [notice that both integrands in (30) behave like $e^{-Hp|q|}$ as $|q| \to \infty$ since $a_+, a_- \to -iq$, $T \to -2q^2$ and $D \to i2(m^2-1)(m_-^2 - m_+^2)q^3$ as $|q| \to \infty$], (3) we employ the fact that as q approaches the negative [positive] real axis of the q-plane in the case $x \geqslant 0 [x \leqslant 0]$ it is valid

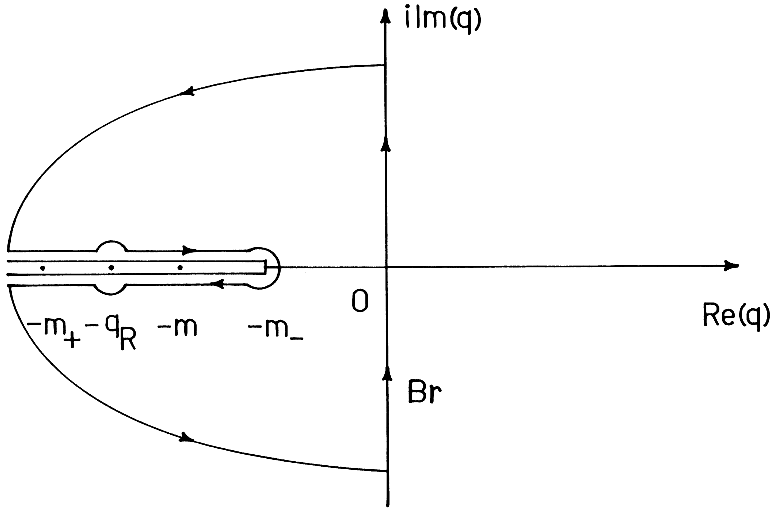

Fig. 3. Contour integration for evaluation of the integrals I_1 and I_2 in eqns (30).

that $q \to -(s/x)$ [of course, this constitutes a basic argument in the Cagniard (1939)–de Hoop (1960) technique]. Finally, we note that in (31) below the contribution from the pole at $-q_R$ is not included. It can be easily seen that this contribution provides a term in the form of a delta Dirac, $\delta(q_R x - v, t)$, in the physical time/space domain.

The previous considerations allows us to write

$$
\begin{aligned}
&U_y(x, y = -H, p) \\
&= \frac{Q\kappa V_1}{\pi p x}\left\{\int_{m_- x}^{mx} \frac{\{[\sin(|a_-|\cdot Hp)\cdot\Pi + \cos(|a_-|\cdot Hp)\cdot\Phi]\cdot|a_+| + \Pi\cdot|a_-|\cdot \mathrm{e}^{-|a_+|\cdot Hp}\}\cdot T}{\Pi^2+\Phi^2}\cdot \mathrm{e}^{-ps}\,\mathrm{d}s\right. \\
&+\int_{mx}^{m_+ x} \frac{\{[\sin(|a_-|\cdot Hp)\cdot K + \cos(|a_-|\cdot Hp)\cdot\Lambda]\cdot|a_+| + K\cdot|a_-|\cdot \mathrm{e}^{-|a_+|\cdot Hp}\}\cdot T}{K^2+\Lambda^2}\cdot \mathrm{e}^{-ps}\,\mathrm{d}s \\
&\left.-\int_{m_+ x}^{\infty} \frac{[\sin(|a_+|\cdot Hp)\cdot|a_-| - \sin(|a_-|\cdot Hp)\cdot|a_+|]\cdot T}{\Psi}\cdot \mathrm{e}^{-ps}\,\mathrm{d}s\right\}, \qquad (31)
\end{aligned}
$$

where

$$T = m^2 - 2(s/x)^2, \qquad (32a)$$

$$\Pi = -|a_+|\cdot M_+ T^2, \qquad (32b)$$

$$\Phi = 4\cdot|a_+|\cdot|a_-|\cdot|\beta|\cdot(s/x)^2(M_- - M_+) + |a_-|\cdot M_- T^2, \qquad (32c)$$

$$K = -4\cdot|a_+|\cdot|a_-|\cdot|\beta|\cdot(s/x)^2(M_- - M_+) - |a_+|\cdot M_+ T^2, \qquad (32d)$$

$$\Lambda = |a_-|\cdot M_- T^2, \qquad (32e)$$

$$\Psi = |a_-|\cdot[-4\cdot|a_+|\cdot|\beta|\cdot(s/x)^2\cdot(M_- - M_+) + M_- T^2] - |a_+|\cdot M_+ T^2, \tag{32f}$$

with

$$|a_+| = (|m_+^2 - (s/x)^2|)^{1/2}, \quad |a_-| = (|m_-^2 - (s/x)^2|)^{1/2}, \quad |\beta| = (|m^2 - (s/x)^2|)^{1/2}, \tag{33a,b,c}$$

and with (m_+, m_-) and (M_+, M_-) being given in (16) and (17), respectively.

Notice in (31) that the second integral is to be interpreted as a Cauchy principal-value integral because of the pole at $s = q_R x$. This observation leads us to conclude that thermoelastic Rayleigh waves will appear at the surface $y = -H$ upon the arrival of which the vertical displacement will become unbounded. Further, this argument can be supported by the following asymptotic considerations, which allow for an approximation of the vertical displacement in the original time/space domain, $u_y(x, y = -H, t)$, valid for long times. For short times, we perform a numerical one-sided Laplace-transform inversion by following the Stehfest (1970) technique.

We start with the asymptotic approach and consider the case when $s \equiv V_1 t$ is large with respect to H ($s > 100H$, say). Then we may take the approximate forms in (21b) with $p \to (1/s)$. These permit writing the following expression for the one-sided Laplace transformed vertical-displacement at the surface

$$U_y(x, y = -H, p) = \frac{Q\kappa V_1}{\pi}\int_{\bar{m}_- x}^{\infty} G(s,x)\cdot e^{-ps}\,ds, \tag{34}$$

where

$$G(s,x) = \frac{sT}{x(\bar{\Pi}^2 + \bar{\Phi}^2)}[\sin(|\bar{a}_-|\cdot(H/s))\cdot|\bar{a}_+|\cdot\bar{\Pi} + \cos(|\bar{a}_-|\cdot(H/s))\cdot|\bar{a}_+|\cdot\bar{\Phi} + |\bar{a}_-|\cdot\bar{\Pi}\cdot\exp(-|\bar{a}_+|\cdot(H/s))] \quad \text{for } \bar{m}_- x < s < mx, \tag{35a}$$

$$G(s,x) = \frac{sT}{x(\bar{K}^2 + \bar{\Lambda}^2)}[\sin(|\bar{a}_-|\cdot(H/s))\cdot|\bar{a}_+|\cdot\bar{K} + \cos(|\bar{a}_-|\cdot(H/s))\cdot|\bar{a}_+|\cdot\bar{\Lambda} + |\bar{a}_-|\cdot\bar{K}\cdot\exp(-|\bar{a}_+|\cdot(H/s))] \quad \text{for } mx < s < \bar{m}_+ x, \tag{35b}$$

$$G(s,x) = \frac{sT}{x\bar{\Psi}}[\sin(|\bar{a}_-|\cdot(H/s))\cdot|\bar{a}_+| - \sin(|\bar{a}_+|\cdot(H/s))\cdot|\bar{a}_-|] \text{for } \bar{m}_+ x < s < \infty, \tag{35c}$$

with $\bar{\Pi}$, $\bar{\Phi}$, $\bar{K}$, $\bar{\Lambda}$ and $\bar{\Psi}$ given by (32) but now with $(|\bar{a}_+|, |\bar{a}_-|)$, $(\bar{m}_+, \bar{m}_-)$ and $(\bar{M}_+, \bar{M}_-)$ replacing the respective quantities there without an overbar. The new quantities with the overbar are defined as

$$|\bar{a}_+| = (|\bar{m}_+^2 - (s/x)^2|)^{1/2}, \quad |\bar{a}_-| = (|\bar{m}_-^2 - (s/x)^2|)^{1/2}, \tag{36a,b}$$

$$\bar{m}_+ = \left[\frac{(1+\varepsilon)s}{h}\right]^{1/2}, \quad \bar{m}_- = \frac{1}{(1+\varepsilon)^{1/2}}, \tag{37a,b}$$

$$\bar{M}_+ = \bar{m}_+^2 - 1, \quad \bar{M}_- = \bar{m}_-^2 - 1. \tag{38a,b}$$

Now, eqn (34) has the form of a recognizable direct one-sided Laplace transform, so the inversion is obtained immediately as

$$u_y(x, y = -H, t \equiv s/V_1) = \frac{Q\kappa V_1}{\pi} \cdot G(s, x) \cdot U(s - \bar{m}_- x), \tag{39}$$

where one may observe that proceeding to the final step of the Cagniard–de Hoop technique [i.e. the one-sided Laplace-transform inversion by inspection which allows getting (39) from (34)] was made possible in the present thermoelastic problem only by the use of asymptotic arguments. This, of course, restricts the validity of the present solution in eqn (39) only for long times.

A graph of the function $G(s, x) = u_y(x, y = -H, t \equiv s/V_1) \cdot (\pi/Q\kappa V_1)$, i.e. the normalized vertical surface displacement, is presented in Fig. 4. This graph was obtained under the restriction $(s/H) \geqslant 100$ and for a material with Poisson's ratio $\nu = 0.20$ (a value which yields $m \equiv V_1/V_2 = 1.632993$), coupling constant $\varepsilon = 0.01$ and thermoelastic characteristic length $h = 10^{-10}$ m. Accordingly, in the pure elastic (non-thermal) case, the velocity of Rayleigh waves

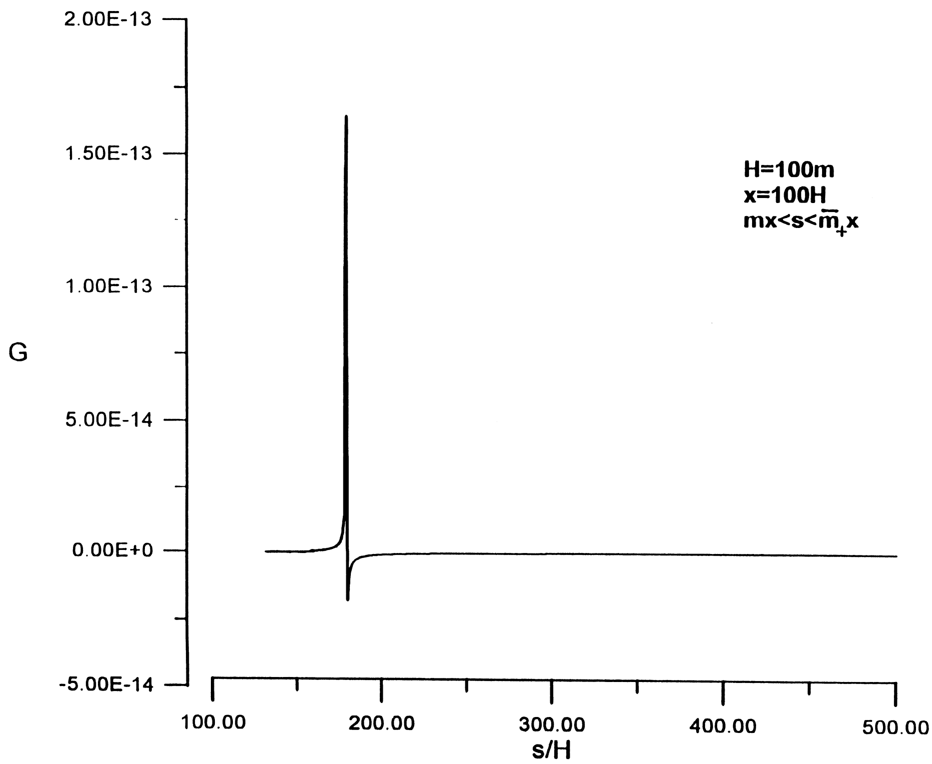

Fig. 4. Variation of the normalized vertical surface displacement G, as defined in eqn (35b), with the normalized time $(s = V_1 t)/H$ for $H = 100$ m, $x = 100H$, $\nu = 0.20$, $\varepsilon = 0.01$ and $h = 10^{-10}$ m.

was found to be $(V_R/V_2) = 0.91$ from Table 7.5.1 of Eringen and Suhubi (1975) and, thus, $q_R \equiv V_1/V_R = 1.79$. It is therefore anticipated that the critical time at which thermoelastic Rayleigh disturbances reach the observation point at the surface will be close to the value $(s/x) = 1.79$. This behavior is indeed observed in Fig. 4, where the case $H = 100$ m, $x = 100H$ and $mx < s < \bar{m}_+ x$ is considered (this case, for instance, could be useful in modeling sub-surface nuclear explosions).

The physical reason for the generation of the strong surface motion (Rayleigh disturbance) observed in the graph of Fig. 4 is the incidence of cylindrical thermoelastic disturbances, emanating from the source point ($x = 0$, $y = 0$), upon the half-plane surface which is insulated and stress-free. A physically analogous mechanism that generates Rayleigh waves is the incidence of a plane pulse upon the curved stress-free surface of a cavity in an elastic medium (Miklowitz, 1964; Norwood and Miklowitz, 1967). As pointed out in the latter studies, this situation is tantamount to a sudden application of the surface of a concentrated loading. Therefore, based on the classical Lamb's (1904) analysis (see also Achenbach, 1973; Eringen and Suhubi, 1975; Miklowitz, 1978) one should expect the generation of strong Rayleigh-wave motions along the surface. Finally, it should also be noted that analogous Rayleigh disturbances appear in Garvin's (1956) dilatational source problem for large time.

Next, we focus attention to a numerical approach for obtaining results for the vertical surface displacement. More specifically, we employ the Stehfest (1970) algorithm to perform the one-sided Laplace-transform inversion (11b) of the exact $U_y(x, y = -H, p)$-expression in eqn (31). This approximate technique, which is particularly suitable when the variable p is taken to be real, was recommended by the well-known survey study on Laplace-transform inversion techniques of Davies and Martin (1979) and has extensively been utilized in transient crack and stress-concentration problems (see e.g. Ang, 1988; Rajapakse and Gross, 1995; Georgiadis et al., 1997). The approximate inversion is given as

$$\varphi(s) \cong \left(\frac{\ln 2}{s}\right) \cdot \sum_{n=1}^{N} c_n \cdot \Phi\left(n \frac{\ln 2}{s}\right), \tag{40}$$

where

$$c_n = (-1)^{n+N/2} \sum_{k=[(n+1)/2]}^{\min(n,N/2)} \frac{k^{N/2}(2k)!}{(N/2-k)!k!(k-1)!(n-k)!(2k-n)!}, \tag{41}$$

N is even (with $N = 18$ good convergence was generally found in the present calculations) and [ ] in (41) denotes the integer part of a number. Of course, with such a numerical technique, one may obtain reliable results only for a limited time-interval and not for the whole time domain. This is due to the inevitable instability of the first-kind integral eqn (11a) for $\varphi(s)$. Nevertheless, we can obtain in this manner useful (although approximate) results, which cannot be revealed by exact analysis. The graphs in Figs 5–7 depict the variation of the normalized vertical surface displacement $u_y(x, y = -H, t) \cdot (\pi/Q\kappa V_1)$ with the normalized time $(V_1/t)/H$ for $H = 100$ m, $x = 0.1H$, $1.0H$ and $10.0H$, and for material constants $v = 0.20$, $\varepsilon = 0.01$, and $h = 10^{-10}$ m. The graphs in Figs 6 and 7, i.e. the graphs which correspond at stations far from the epicenter, show again the generation of strong Rayleigh motions at the half-space surface. On the opposite, the graph in Fig. 5 shows the generation of a complicated pattern of vertical motions near the epicenter. In addition, the graph of Fig. 8 is presented which was obtained for the case $H = 10$ m and $x = 0.1H$, and for the

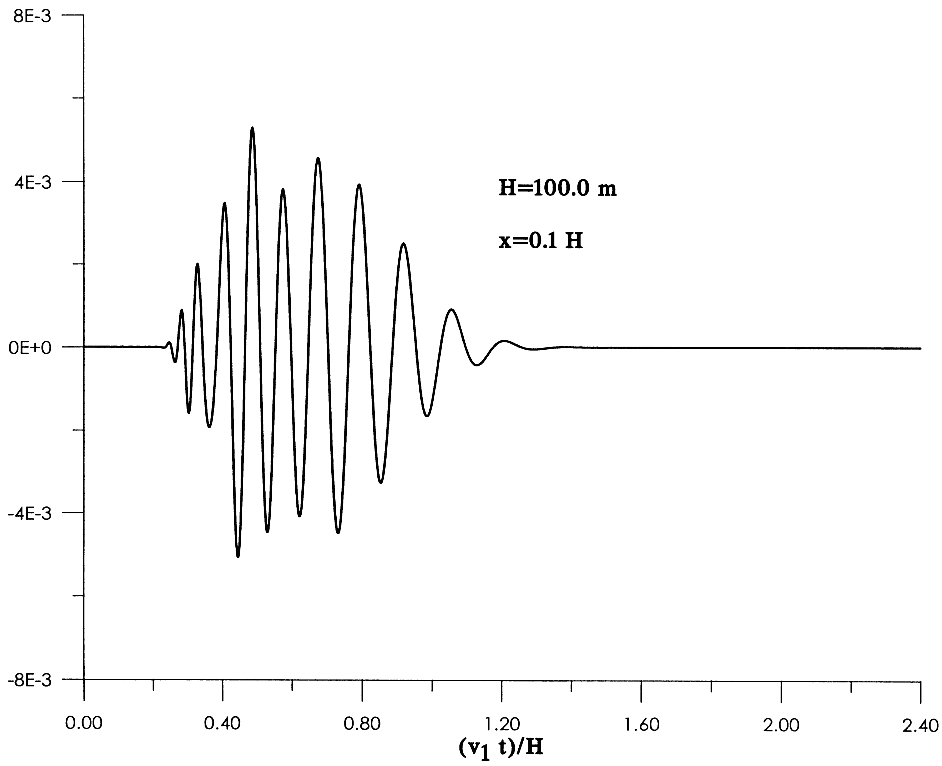

Fig. 5. Variation of the normalized vertical surface displacement $u_y(x, y = -H, t)(\pi/Q\kappa V_1)$ with the normalized time $(s = V_1 t)/H$ for $H = 100$ m, $x = 0.1H$, $\nu = 0.20$, $\varepsilon = 0.01$ and $h = 10^{-10}$ m.

same material constants as before. The history of the normal displacement at the surface resembles the one in Fig. 5 (which again corresponds to a station near the epicenter) but here the absolute values of the normalized displacement are increased by one order of magnitude. This is not unexpected because in the case of Fig. 8 the source was placed closer to the half-space surface as compared to the case of Fig. 5.

We close the presentation of results by noticing the behavior of the normal displacement at the half-space surface near the origin, $\lim_{x\to 0} u_y(x, y = -H, t)$, when the thermal source is placed at the surface, i.e. for $H = 0$. In this case, eqn (29) becomes

$$\lim_{H\to 0} U_y(x, y = -H, p) = \frac{Q\kappa V_1}{2\pi i p}\int_{-i\infty}^{i\infty} \frac{(a_+ - a_-)T}{D} e^{pqx}\, dq. \tag{42}$$

Then, an estimate for the displacement near $x = 0$ is provided by the asymptotic expression of the integrand in (42) for $|q| \to \infty$ (van der Pol and Bremmer, 1950). By noticing that $(a_+ - a_-) \to (i/2q)(m_+^2 - m_-^2)$, $T \to -2q^2$ and $D \to i2q^3(m^2-1)(m_-^2 - m_+^2)$ for $|q| \to \infty$, we get

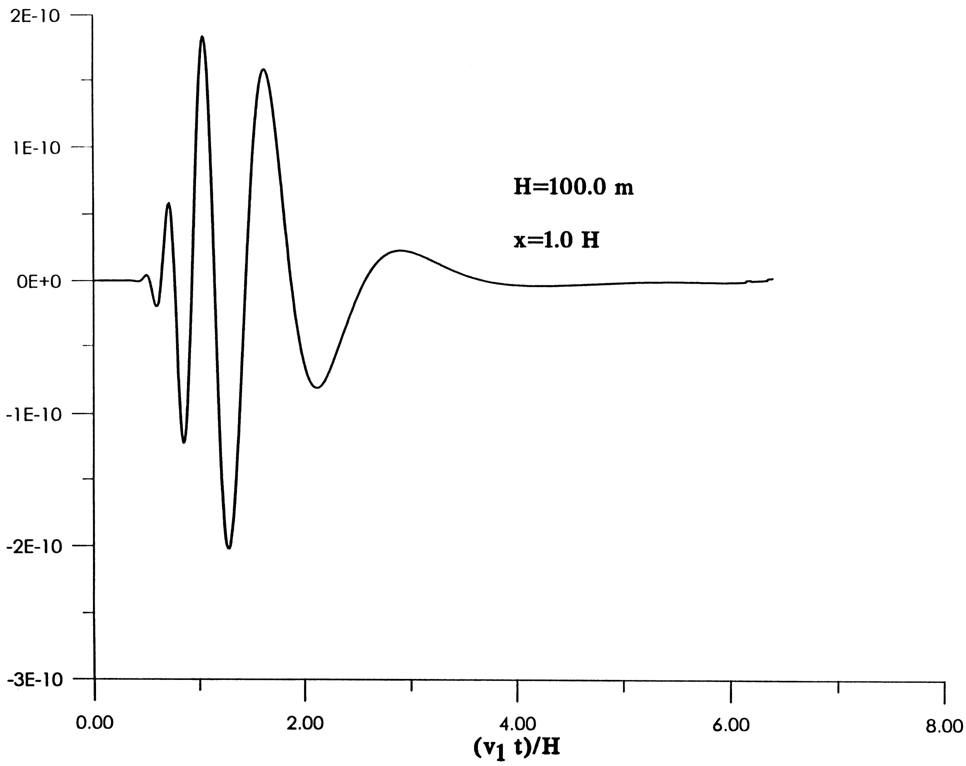

Fig. 6. Variation of the normalized vertical surface displacement $u_y(x, y = -H, t)(\pi/Q\kappa V_1)$ with the normalized time $(s = V_1 t)/H$ for $H = 100$ m, $x = 1.0H$, $v = 0.20$, $\varepsilon = 0.01$ and $h = 10^{-10}$ m.

$\lim_{|q|\to\infty} [(a_+ - a_-)T/D] = [2(m^2-1)q^2]^{-1}$, which when inverted (Gel'fand and Shilov, 1964) provides

$$\lim_{\substack{H\to 0\\ x\to 0}} U_y(x, y = -H, p) = -\frac{Q\kappa V_1}{4(m^2-1)}|x|, \tag{43}$$

and

$$\lim_{\substack{H\to 0\\ x\to 0}} u_y(x, y = -H, t) = \frac{Q\kappa}{4(m^2-1)}|x| \cdot \delta(t). \tag{44}$$

The above result for the normal surface displacement clearly shows a discontinuity generated at the point of application of the surface thermal source. It is therefore anticipated that surface waves (thermoelastic Rayleigh waves) will be generated in this case too as in the case of a buried thermal source.

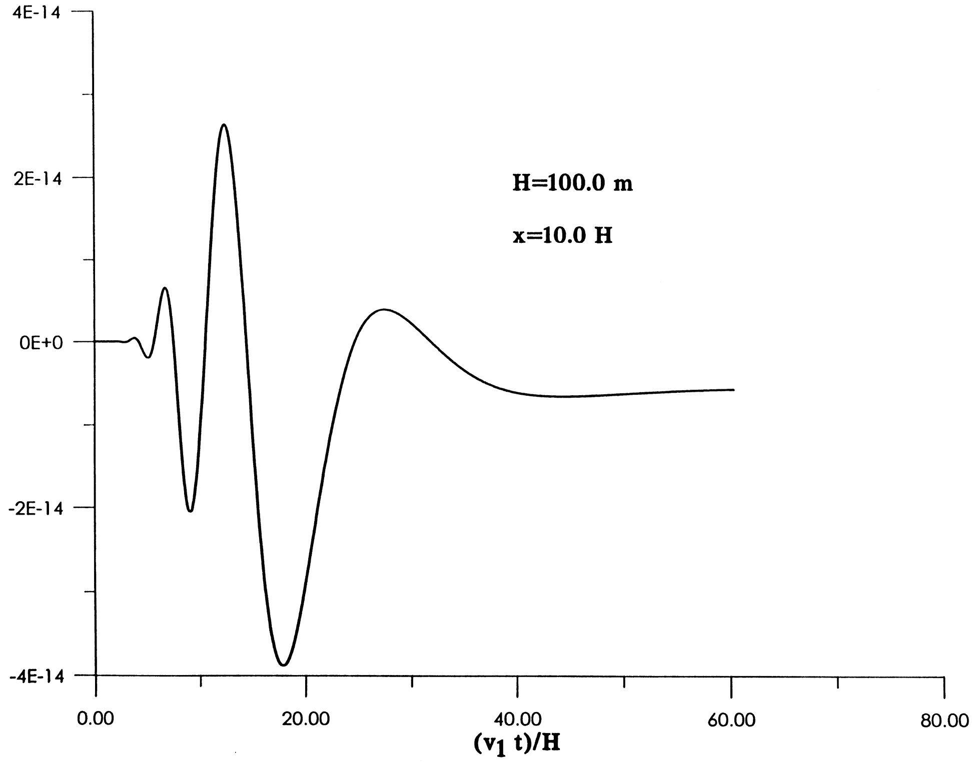

Fig. 7. Variation of the normalized vertical surface displacement $u_y(x, y = -H, t)(\pi/Q\kappa V_1)$ with the normalized time $(s = V_1 t)/H$ for $H = 100$ m, $x = 10.0H$, $v = 0.20$, $\varepsilon = 0.01$ and $h = 10^{-10}$ m.

5. Conclusions

The 2-D transient dynamic coupled-thermoelastic problem of a buried thermal/mechanical source in a half-space medium was treated in the present work. The problem was intended to model underground nuclear explosions and impulsively applied heat loading near a boundary, and the present solution provides the Green's function for more general spatial/temporal loadings. Our problem can be viewed as a generalization of the classical Nakano–Lapwood–Garvin and Pekeris–Payton–Tsai and Ma problems in that of considering additional coupled-thermoelastic constitutive behavior for the medium and an additional thermal loading. Representative numerical results were given for the vertical surface–displacement under a thermal loading with a Dirac delta-type variation in time. An asymptotic procedure, which is fairly adequate in some instances, and a numerical approach were employed to invert the one-sided (time) Laplace-transformed displacement. The latter, however, had earlier been obtained through exact analysis. The results clearly demonstrate the dominance of the thermoelastic Rayleigh wave at large distances from the epicenter.

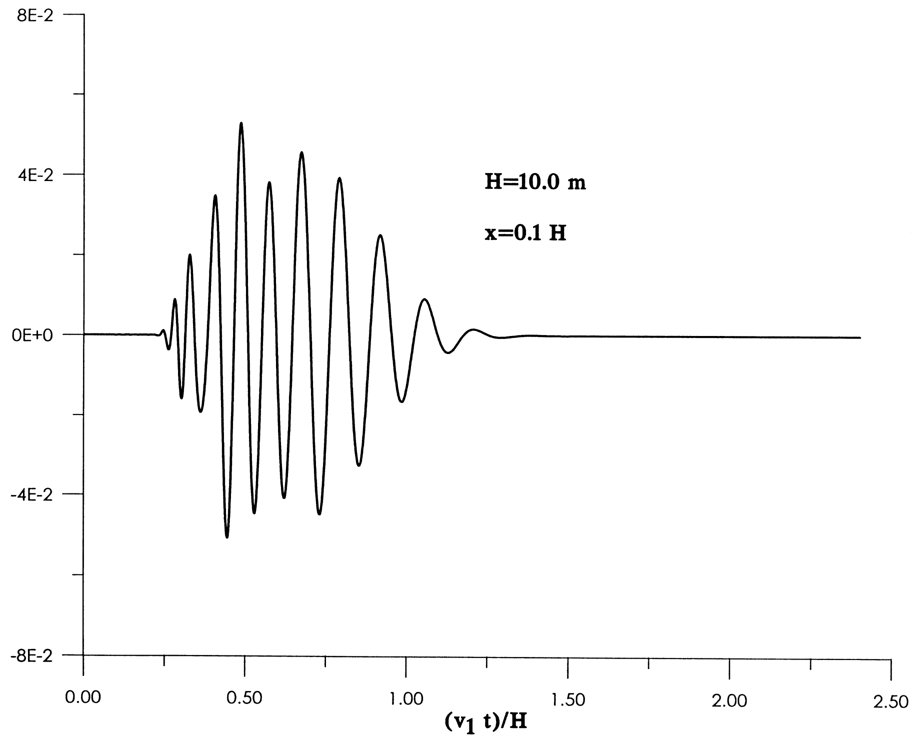

Fig. 8. Variation of the normalized vertical surface displacement $u_y(x, y = -H, t)(\pi/Q\kappa V_1)$ with the normalized time $(s = V_1 t)/H$ for $H = 10$ m, $x = 0.1H$, $v = 0.20$, $\varepsilon = 0.01$ and $h = 10^{-10}$ m.

Acknowledgements

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Appendix A

The 9×9 system of Section 3 reads

$$X_1 + X_2 - \frac{1}{q} X_3 - X_4 - X_5 - X_6 - X_7 + \frac{1}{q} X_8 + \frac{1}{q} X_9 = 0, \tag{A1a}$$

$$X_1 + \frac{a_-}{a_+} X_2 + \frac{q}{a_+ \beta} X_3 + X_4 - X_5 + \frac{a_-}{a_+} X_6 - \frac{a_-}{a_+} X_7 + \frac{q}{a_+ \beta} X_8 - \frac{q}{a_+ \beta} X_9 = 0, \tag{A1b}$$

$$X_1 + \frac{M_-}{M_+} X_2 - X_4 - X_5 - \frac{M_-}{M_+} X_6 - \frac{M_-}{M_+} X_7 = 0 \tag{A1c}$$

$$X_1+X_2+\frac{2q}{T}X_3-X_4-X_5-X_6-X_7-\frac{2q}{T}X_8-\frac{2q}{T}X_9=-\frac{PV_1}{\mu T}, \tag{A1d}$$

$$X_1+\frac{a_-}{a_+}X_2-\frac{T}{2qa_+\beta}X_3+X_4-X_5+\frac{a_-}{a_+}X_6-\frac{a_-}{a_+}X_7-\frac{T}{2qa_+\beta}X_8+\frac{T}{2qa_+\beta}X_9=-\frac{SV_1}{2\mu qa_+}, \tag{A1e}$$

$$X_1+\frac{a_-M_-}{a_+M_+}X_2+X_4-X_5+\frac{a_-M_-}{a_+M_+}X_6-\frac{a_-M_-}{a_+M_+}X_7=-\frac{Q\kappa V_1}{m^2pa_+M_+}, \tag{A1f}$$

$$(pa_+M_+\,\mathrm{e}^{-pa_+H})X_4-(pa_+M_+\,\mathrm{e}^{pa_+H})X_5+(pa_-M_-\,\mathrm{e}^{-pa_-H})X_6-(pa_-M_-\,\mathrm{e}^{pa_-H})X_7=0, \tag{A1g}$$

$$\begin{aligned}&-(2qa_+\,\mathrm{e}^{-pa_+H})X_4+(2qa_+\,\mathrm{e}^{pa_+H})X_5-(2qa_-\,\mathrm{e}^{-pa_-H})X_6\\&\qquad+(2qa_-\,\mathrm{e}^{pa_-H})X_7+\left(\frac{T}{\beta}\mathrm{e}^{-p\beta H}\right)X_8-\left(\frac{T}{\beta}\mathrm{e}^{p\beta H}\right)X_9=0,\end{aligned} \tag{A1h}$$

$$\begin{aligned}&-(T\,\mathrm{e}^{-pa_+H})X_4-(T\,\mathrm{e}^{pa_+H})X_5-(T\,\mathrm{e}^{-pa_-H})X_6\\&\qquad-(T\,\mathrm{e}^{pa_-H})X_7-(2q\,\mathrm{e}^{-p\beta H})X_8-(2q\,\mathrm{e}^{p\beta H})X_9=0,\end{aligned} \tag{A1i}$$

and it has the solution

$$\begin{aligned}X_1=&\frac{V_1M_-T^2(Pa_-M_+P-SM_+pq+Q\kappa\mu)\,\mathrm{e}^{-(a_++a_-)Hp}}{\mu m^2p(M_--M_+)D}\\&-\frac{2V_1a_-M_-qT(S\beta+Pq)\,\mathrm{e}^{-(a_++\beta)Hp}}{\mu m^2D}\\&+\frac{V_1(Pa_+M_-p-SM_-pq+Q\kappa\mu)E\,\mathrm{e}^{-2a_+Hp}}{2a_+m^2\mu p(M_--M_+)D}\\&-\frac{V_1(Pa_+M_-p+SM_-pq-Q\kappa\mu)}{2a_+m^2\mu p(M_--M_+)},\end{aligned} \tag{A2a}$$

$$\begin{aligned}X_2=&\frac{V_1M_+T^2(Pa_+M_-p-SM_-pq+Q\kappa\mu)\,\mathrm{e}^{-(a_++a_-)Hp}}{\mu m^2p(M_--M_+)D}\\&+\frac{2V_1a_+M_-qT(S\beta+Pq)\,\mathrm{e}^{-(a_-+\beta)Hp}}{\mu Dm^2}\\&-\frac{V_1(Pa_-M_+p+Q\kappa\mu-SM_+pq)F\,\mathrm{e}^{-2a_-Hp}}{2a_-m^2\mu pD(M_--M_+)}\\&+\frac{V_1(Pa_-M_+p+SM_+pq-Q\kappa\mu)}{2a_-m^2\mu p(M_--M_+)},\end{aligned} \tag{A2b}$$

$$X_3 = \frac{2V_1 a_+ \beta q T(Pa_- M_+ p - SM_+ pq + Q\kappa\mu)\,\mathrm{e}^{-(a_- + \beta)Hp}}{m^2 \mu p D} - \frac{2V_1 a_- \beta q T(Pa_+ M_- p - SM_- pq + Q\kappa\mu)\,\mathrm{e}^{-(a_+ + \beta)Hp}}{m^2 \mu p D} - \frac{V_1 (S\beta + Pq) J\,\mathrm{e}^{-2\beta Hp}}{2\mu D m^2} + \frac{V_1 (S\beta - Pq)}{2\mu m^2}, \tag{A2c}$$

$$X_4 = \frac{V_1 (Pa_+ M_- p - SM_- pq + Q\kappa\mu)}{2m^2 \mu a_+ p(M_- - M_+)}, \tag{A2d}$$

$$X_5 = \frac{V_1 M_- T^2 (Pa_- M_+ p - SM_+ pq + Q\kappa\mu)\,\mathrm{e}^{-(a_+ + a_-)Hp}}{m^2 \mu p D} - \frac{V_1 M_- T[4a_- \beta q T(Pa_+ M_- p - SM_- pq + Q\kappa\mu)\,\mathrm{e}^{-(a_+ + \beta)Hp} + p(S\beta + Pq) J\,\mathrm{e}^{-2\beta Hp}]\,\mathrm{e}^{-(a_+ - \beta)Hp}}{4m^2 \mu a_+ \beta p q D(M_- - M_+)} - \frac{V_1 M_- T(S\beta + Pq)\,\mathrm{e}^{-(a_+ + \beta)Hp}}{4\mu m^2 a_+ \beta q(M_- - M_+)} + \frac{V_1 (Pa_+ M_- p - SM_- pq + Q\kappa\mu)\,\mathrm{e}^{-2a_+ Hp}}{2m^2 \mu a_+ p(M_- - M_+)}, \tag{A2e}$$

$$X_6 = -\frac{V_1 (Pa_- M_+ p - SM_+ pq + Q\kappa\mu)}{2m^2 \mu a_- p(M_- - M_+)}, \tag{A2f}$$

$$X_7 = \frac{V_1 M_+ T^2 (Pa_+ M_- p - SM_- pq + Q\kappa\mu)\,\mathrm{e}^{-(a_+ + a_-)Hp}}{m^2 \mu p D(M_- - M_+)} + \frac{2V_1 a_+ M_+ q T(S\beta + Pq)\,\mathrm{e}^{-(a_- + \beta)Hp}}{\mu D m^2} - \frac{V_1 (Pa_- M_+ p - SM_+ pq + Q\kappa\mu) F\,\mathrm{e}^{-2a_- Hp}}{2m^2 \mu \alpha_- p D(M_- - M_+)}, \tag{A2g}$$

$$X_8 = \frac{V_1 (S\beta + Pq)}{2\mu m^2}, \tag{A2h}$$

$$X_9 = \frac{2V_1 \alpha_+ \beta q T(Pa_- M_+ p - SM_+ pq + Q\kappa\mu)\,\mathrm{e}^{-(a_- + \beta)Hp}}{m^2 \mu p D} - \frac{2V_1 a_- \beta q T(Pa_+ M_- p - SM_- pq + Q\kappa\mu)\,\mathrm{e}^{-(\alpha_+ + \beta)Hp}}{m^2 \mu p D} - \frac{V_1 (S\beta + Pq) J\,\mathrm{e}^{-2\beta Hp}}{2\mu D m^2}, \tag{A2i}$$

where

$$D \equiv D(q,p) = a_-[4\alpha_+\beta q^2(M_- - M_+) + M_- T^2] - \alpha_+ M_+ T^2, \tag{A3}$$

$$E \equiv E(q,p) = a_-[4\alpha_+\beta q^2(M_- - M_+) - M_- T^2] - \alpha_+ M_+ T^2, \tag{A4}$$

$$F \equiv F(q,p) = a_-[4\alpha_+\beta q^2(M_- - M_+) + M_- T^2] + \alpha_+ M_+ T^2, \tag{A5}$$

$$J \equiv J(q,p) = a_-[4\alpha_+\beta q^2(M_- - M_+) - M_- T^2] + \alpha_+ M_+ T^2. \tag{A6}$$

Appendix B

$$p \cdot U_y^*(q, y = -H, p) = -a_+ X_4\, e^{-a_+ Hp} + a_+ X_5\, e^{a_+ Hp} - a_- X_6\, e^{-a_- Hp} + a_- X_7\, e^{a_- Hp} - \frac{q}{\beta} X_8\, e^{-\beta Hp} + \frac{q}{\beta} X_9\, e^{\beta Hp}. \tag{B1}$$

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